

Circular Functions – Discovering the Unit Circle

R. Davis, Sandia High School, Albuquerque, NM

Adapted from activity by Ron Stewart, Baylor School, Chattanooga, TN

MATERIALS NEEDED: compass, protractor, measuring sticks (unmarked strips of paper), graphing calculators

1. **Without** using a compass, find an object that can be used to trace the outline of a circle. Your object should make as large a circle as possible on the 11 x 14 sheet of construction paper you will be given.
2. Carefully trace the outline of your object on the paper. Use a pencil for your first outline in case you make a mistake. After you have drawn a good circle, you can trace over your drawing using a pen or fine-tipped marker. What item did you use to trace? _____
3. Using a straight edge and a compass, find the center of your circle. Draw your lines lightly (and do not erase them), as you will be taking measurements from your circle later.
Hint: What theorems do you know about chords of circles that may help you?
4. Draw a diameter of your circle.
5. Construct another diameter for your circle, perpendicular to the one you drew in #4. These diameters will intersect at the center of the circle.
6. Extend the two diameters that you drew beyond the circle. In effect, you have constructed two coordinate axes. Label one of them the x-axis and the other one the y-axis. Lightly label each of the four quadrants (I, II, III and IV) on your paper.
7. Make a measuring system of your own. Create your own measuring stick (be sure that it is longer than the radius of your circle). Make it as accurate as possible. Make up a name for your units. For example, let one unit be one gum wrapper. Now, you will be measuring in lengths of gum wrappers instead of inches or centimeters. What is the radius of your circle in terms of your new unit? _____ What is the equation of your circle in terms of your new unit? _____
(Note: The equation of a circle centered at the origin is $x^2 + y^2 = r^2$)
8. Decide on a method that will divide the circumference of your circle into 36 equally spaced segments (arcs). You may use a protractor and your new measuring stick to do this. What is the measure of each of the central angles which have been formed by these arcs? _____ Mark each of these dividing points.
9. Enter the numbers {0, 10, 20, 30, ..., 350} into list 1 in your calculator. (STAT EDIT
SetUpEditor Enter . . . you will see Done on the homescreen of your calculator. Next, go to STAT EDIT and enter your angles into list 1)

Why was 360 not entered into the list? _____
10. Using your new units, determine the x and y-coordinates of each of the points that you marked in #8. This may take a while, and can be quite tedious, but *be as accurate as you can, as this is one of the most crucial aspects of this activity*. Record the x-coordinates in list 2 and the y-coordinates into list 3. (NOTE: These coordinates must correspond to each of the angle measurements you have entered in L1)
11. The number of elements in each list is called the dimension of the list. Check to see that each list (list 1, 2 and 3) all have the same dimensions – that is 36 entries in each list, and NOT 37.

fix C1, L2, L3, L4, L5
A B C D E

12. What is the radius of your circle (in your new units)? _____ This is the same value you listed in #7, and will be denoted as R . Highlight list 4 in your calculator and enter L_2/R (that is, enter second 2 to get L_2 and enter the actual number for your radius). You should see a new set of numbers appear in L_4 . Now, highlight L_5 , and enter L_3/R . You should see a different set of numbers appear in this list. What do the numbers in L_4 represent? _____ In L_5 ? _____

13. You are now ready to construct a scatter plot of your data. In the Stat Plot menu, turn on Plot 1 and select the first type of scatter plot that is given. In the xlist, enter L_1 and in the ylist enter L_2 . Choose the dot as the mark.

14. Set a window appropriate to your data.

15. Press GRAPH to view your scatter plot. Sketch your graph below (or obtain a printout), indicating units on both sets of axes. Also, be sure to indicate all important features (maxima, minima, intercepts, etc) on your graph.

Sketch your graph here:

16. Now find 2 other people who have a radius quite a bit different than yours. Ideally, you will want one person's radius who is much smaller than yours, and another person's radius who is much larger than yours. Link your calculators, one at a time, to obtain each other's $L_2, L_3, L_4, & L_5$. Make sure you do not overwrite your lists, but instead rename the others group L_2 as BL2 (for example, Brittany's list 2) and BL3 (for Brittany's list 3 or something else you will remember), and so forth until you have 4 additional lists for 2 other people. To name a list something other than its given name of L_1, L_2, \dots , highlight L_1 and right arrow over until you see a list marked as ----- . You can now name this list with alpha keys. Create 3 scatter plots in the same window, one with your original data (using dots) and the other two with the other 2 persons data (each scatter plot will have an xlist of L_1 in which everyone should have the same numbers and a ylist of the x-coordinates of that person's circle. To input that list of data into your scatterplot, you must go to 2nd STAT and arrow down until you see the name of the list you have renamed and then press enter). Use the square for one person's data and the plus for the other person's data. Reset your window. Add the 2 new graphs to your graph in #15, carefully labeling all important parts of the graph and each of the axes. Color code the other 2 graphs, and make a key.

Make some conjectures about the data (that is, the 3 graphs in #15): (note similarities and differences)

17. Repeat procedures 13 – 16, using L_3 in place of L_2 . Draw the graph of all 3 L_3 vs L_1 plots below. Make some conjectures about this data. Use the same color coding as in #15. Also, be sure to label all important parts of the graphs (intercepts, max & mins) as well as scaling your axes.

18. Do the same for scatter plots of L_4 vs. L_1 and L_5 vs. L_1 , repeating steps # 13 – 16.
Graph of L_4 vs. L_1 : Again, use the same color coding of the 3 different graphs.

Conjectures about the data of L_4 vs. L_1 :

Graph of L_5 vs. L_1 : You again will have 3 graphs here . . . yours, and the 2 other persons you collected.

Conjectures about the data of L_5 vs. L_1 :

19. Now compare all of your graphs. Spend some time looking at your graphs and noting the similarities and differences to your original circle. Note those here:

20. Make sure your final graph of your circle is clearly marked and labeled. Be sure all your graphs are labeled and clearly marked as your data vs person 1 vs person 2. Attach your measuring stick with your graphs.

Grading Rubric:

1. _____ Construction of Circle (20 points)
 - (1) Circle drawn is large enough to read all information
 - (4) Accurate, non-standard measuring stick made
 - (2) Correct center of circle found – with appropriate geometric constructions shown
 - (2) Correct axes drawn – with appropriate geometric constructions shown
 - (8) Accurate measurements of x and y-coordinates
 - (3) Circle is neat and readable

2. _____ Data Analysis (20 points)
 - (12) Well-constructed and correct scatter plots
 - L2 vs. L1
 - L3 vs. L1
 - L4 vs. L1
 - L5 vs. L1
 - (8) All axes are labeled and scaled appropriately on the scatter plots

3. _____ Completion of Project (10 points)
 - (8) All conjectures on this handout are completed
 - (2) Project posted with circle, all graphs and measuring stick attached

TOTAL SCORE: _____